THETA CIPHERING ANSWERS

0. 54

1. $\frac{152\pi}{15}$

2. (-1.5, 0)

3. $\frac{169}{24}$ *or* $7\frac{1}{24}$

4. 4

5. $\begin{bmatrix} 3 & 0 \\ -8 & -7 \end{bmatrix}$

6. $72\sqrt{3} - 36\pi$

7. 1

8. -5, -1, 4

9. $\frac{1+3\sqrt{5}}{2}$

10. $\left(\frac{2}{3}, 1\right) \cup (1, 2) \cup (3, \infty)$

11. $\frac{x^2+1}{x^3+2x}$

12. 128

13. 4

14. $\frac{7}{2}$

2018 THETA CIPHERING SOLUTIONS

0.	54	$3 \cdot 6 \cdot 3$ = 3 choices for hundreds place, 6 choices for tens place, 3 even digits for ones place.
1.	$\frac{152\pi}{15}$	Arc length is a fraction (38/60) of the circumference (16π)
2.	(-1.5, 0)	Slope of perpendicular line will be $-\frac{4}{3}$ and the midpoint of the segment is (-3, 2). Using point-slope form $y - 2 = -\frac{4}{3}(x+3)$ leads to $4x + 3y = -6$ and letting y = 0, gives x = -1.5.
3.	$\frac{169}{24}$ *or* $7\frac{1}{24}$	ΔABC is a right triangle by the Pythagorean Theorem. $\Delta AMP \cong \Delta BMP$ by SAS, so AP=PB. Let AP = x. Using the Pythagorean Theorem on ΔAPC, $25 + (12-x)^2 = x^2$ which gives $25 + 144 - 24x + x^2 = x^2$ and so x = 169/24.
4.	4	The center (h, k) is (0, -4) and c = 4. Since a vertex is (0, 1), a = 5. $b^2 = a^2 - c^2$ resulting in b = 3. So, a + b + h + k = 5 + 3 + 0 +(-4) = 4.
5.	$\begin{bmatrix} 3 & 0 \\ -8 & -7 \end{bmatrix}$	$AB = \begin{bmatrix} 6 & 0 \\ -12 & -9 \end{bmatrix}$ and $B^{-1}A = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -4 & -2 \end{bmatrix}$ Subtract the two results and you get $\begin{bmatrix} 3 & 0 \\ -8 & -7 \end{bmatrix}$.
6.	$72\sqrt{3} - 36\pi$	The radius of the circle is 6 so its area is 36π. Using the 30-60-90 triangle formed by drawing a radius of the circle to the point of tangency to the hexagon (an apothem) and a segment from the center to a vertex of the hexagon, the short leg of the triangle is $2\sqrt{3}$. This makes the side of the hexagon $4\sqrt{3}$. The formula for area = $\frac{1}{2}$(apothem)(perimeter) so $A = \frac{1}{2} \cdot 6 \cdot 24\sqrt{3} = 72\sqrt{3}$. The area between them is $72\sqrt{3} - 36\pi$.
7.	1	$4(-1) + 2\left(\frac{3}{2}\right) + 6\left(\frac{1}{3}\right) = -4 + 3 + 2 = 1$
8.	-5, -1, 4	Try the test for -1 as a root of f(-x). It works! Synthetically divide by -1 to get the quotient $x^2 + x - 20 = 0$ which factors to $(x + 5)(x - 4) = 0$ and gives zeros of -5 and 4.

9. $\frac{1+3\sqrt{5}}{2}$ Let $x = \sqrt{11 + \sqrt{11 + \sqrt{11 + \ldots}}}$. Squaring both sides gives $x^2 = 11 + x$ which can be solved by the quadratic formula as $x = \frac{1 \pm 3\sqrt{5}}{2}$. Only the positive solution works.

10. $\left(\frac{2}{3}, 1\right) \cup (1, 2) \cup (3, \infty)$ The argument of the logarithm must be positive which gives the resulting domain of $(-3, 2) \cup (3, \infty)$. The base of the logarithm, 3x – 2, must also be positive and not equal to 1. This means that x must be greater than $\frac{2}{3}$ but not equal to 1.

11. $\frac{x^2+1}{x^3+2x}$ $\left(x + \frac{1}{x}\right)^{-1}$ becomes $\frac{x}{x^2+1}$. $\left[x + \frac{x}{x^2+1}\right]^{-1} = \left(\frac{x^3+2x}{x^2+1}\right)^{-1} = \frac{x^2+1}{x^3+2x}$

12. 2^7 or 128 The prime factorization of 2016 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$. The smallest prime factor is 2 and the largest prime factor is 7.

13. 4 Solving the first inequality gives $-10 \leq 3x - 4 \leq 10$ resulting in $-2 \leq x \leq \frac{14}{3}$. Solving the second gives $3x + 2 > 4$ *or* $3x + 2 < -4$ leading to the solution of $x > \frac{2}{3}$ *or* $x < -2$. The only integers that satisfy both are 1, 2, 3, and 4.

14. $\frac{7}{2}$ Let $m = 2^{2x}$. The equation becomes $3(2^{2x}2^3) - (2^{2x})^2 = 128$ and after substituting we get $24(m) - m^2 = 128$, or $m^2 - 24m + 128 = 0$. This factors into $(m - 16)(m - 8) = 0$ so $m = 8$ or $m = 16$. Solving for x, we get $2^{2x} = 8$ which yields $x = 3/2$ and $2^{2x} = 16$ which yields $x = 2$. The sum is $3/2 + 2 = 7/2$.